Seat Number

नमुना - 010

MATHEMATICS PAPER-II : MTH-102
Calculus
(811102)

P. Pages : 4

Time : Two Hours

Max. Marks : 60

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to right indicate full marks.

1. a) Attempt **any six** of the following. **6**

i) $\lim_{x\to\infty} \frac{\log x}{x}$ is ------

a) 1 b) 0
c) -1 d) ∞

ii) $\lim_{x\to\frac{\pi}{2}} (\sec x - \tan x)$ is -----

a) 0 b) 1
c) -1 d) $\frac{\pi}{2}$

iii) Function $f(x) = |x - a|$ is ----- at $x = a$

a) continuous
b) differentiable
c) continuous but differentiable
d) Neither continuous nor differentiable

iv) Using Rolle's theorem for $f(x) = x^2 - 6x + 5$ in [1, 5] the value of C is---

a) 1 b) 3
c) -6 d) 5

v) If $y=(ax+b)^n$ then $y_n=$.....

a) $n!a^n$ b) $n!$

c) 0 d) None of these

vi) If $y=\sin 5x$ then $y_n=$.....

a) $\sin 5x$ b) $\cos 5x$

c) $5\cos 5x$ d) $5^n \sin\left(5x+n\frac{\pi}{2}\right)$

vii) $1-x+x^2-x^3+x^4.....+(-1)^n x^n+....$ is an expansion of -----

a) $\frac{1}{1-x}$ b) $\frac{1}{1+x}$

c) e^x d) e^{-x}

viii) $\int_0^{\pi} \frac{\sin 9x}{\sin x} dx=$.....

a) π b) 0

c) 9π d) 2π

b) Attempt **any six** of the following. 6

i) Define continuity of a function at a point x = a.

ii) Define supremum of a function.

iii) Is function $f(x)=|x|$ derivable at x = 0?

iv) State alternative form of Lagrange's mean Value Theorem.

v) If $y=\log(ax+b)$ then $y_n=$.....

vi) If $y=\cos x$ then $y_n=$.....

vii) Maclaurin's series expansion of sinx in powers of x is ------

viii) $\int_0^{\pi/2} \cos^7 x\, dx=$.....

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2. Attempt **any six** of the following.

i) Find $\lim_{x\to 0} \frac{e^{-x} - e^{x} + 2x}{x - \sin x}$.

ii) Find $\lim_{x\to 0} \sin x \cdot \log x$.

iii) Define Right hand limit of a function at a point x = 9.

iv) Show that $f(x) = |x - 1|$ is continuous at x = 1.

v) Define Monotonic Increasing function.

vi) Define Successive Differentiation.

vii) Find n^{th} derivative of $e^{x} \cos x$.

viii) Evaluate $\int_0^{\pi} \cos^7 \frac{x}{2} dx$.

ix) Evaluate $\int_0^{\pi/2} \sin^3 x \cos^6 x \, dx$.

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3. Attempt **any four** of the following.

i) Examine the continuity of the function

$$f(x) = \frac{x^2}{4} - 4, \quad \text{for } 0 < x < 4$$
$$= 2, \quad \text{for } x = 4$$
$$= 4 - \frac{64}{x^2}, \quad \text{for } x > 4$$

at point x = 4.

ii) Evaluate $\lim_{x\to 0} (\cos x)^{\cot x}$.

iii) Verify Lagrange's Mean Value Theorem for the function $f(x) = x^2 - 3x + 2$ in $[-2, 3]$.

iv) State and prove Cauchy's Mean Value Theorem.

v) If $y = x^3 \cos x$ then find y_n.

vi) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x-2)$.

4. Attempt **any three** of the following. **12**

i) Show that every continuous function on closed and bounded interval is attains its bounds.

ii) If $f(x) = \dfrac{\sin 2x}{3x} + a$, if $x > 0$

$= 3 + b$, if $x < 0$

$= 3/2$, if $x = 0$

is continuous at point x = 0, then find values of a and b.

iii) Prove that $\dfrac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \dfrac{b-a}{1+a^2}$ if $0 < a < b$.

iv) If $y = e^{ax} \sin(bx + c)$ then prove that

$$y_n = (a^2 + b^2)^{n/2} e^{ax} \sin\left(bx + c + n \tan^{-1} \frac{b}{a}\right)$$

v) Evaluate $\int_0^1 x^{7/2} (1-x)^{5/2} \, dx$

5. Attempt **any two** of the following. **12**

i) State and prove Rolle's Theorem.

ii) If $y = a\cos(\log x) + b\sin(\log x)$ then show that

$x^2 y_{n+2} + (2n+1) x y_{n+1} + (n^2+1) y_n = 0$

iii) Show that $\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx = \dfrac{n-1}{m+n} \int_0^{\pi/2} \sin^m x \cdot \cos^{n-2} x \, dx$.

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