B) Graph Theory (811104)

Time : Two Hours **Max. Marks : 60**

Instructions to Candidates :

1. Do not write anything on question paper except Seat No.
2. Graph or diagram should be drawn with the black ink pen being used for writing paper or black HB pencil.
3. Students should note, no supplement will be provided.
4. All questions are compulsory.
5. Figures to the right indicate full marks.

1. a) Attempt **any six** of the following. **6**

1) Total no. of edges in a complete graph k_7 is

a) 21 b) 28
c) 42 d) 49

2) The complement of the null graph is

a) full graph b) complete graph
c) null graph d) any graph

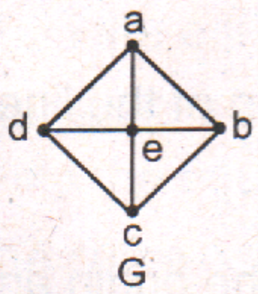
3) Which of the following graph is Eulerian graph

a) $K_{1,4}$ b) $K_{3,3}$
c) $K_{3,4}$ d) $K_{4,4}$

4) If G is simple graph with n-vertices m edges and k components then

a) $m \geq n-k$ b) $m \leq n-k$
c) $m+n-k=0$ d) $k \geq 2$

5) The edge connectivity of the graph G as below is



G

a) 1 b) 2
c) 3 d) 4

6) The chromatic no. of a triangle is

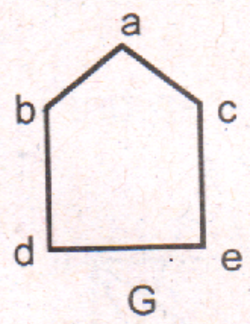
a) 0 b) 1
c) 2 d) 3

7) If G is a graph on p – vertices, q edges and k components then nullity of G is
   a) p – q + k
   b) p – k
   c) q – k
   d) q – p + k

8) Which of the following statement is false
   a) Every tree is bipartite graph
   b) In a tree there is unique path between any two vertices.
   c) The graph having no circuit is always a tree
   d) Every tree has either one or two centres

6

b) Attempt **any six.**
   1) Define degree of a vertex in the graph.
   2) Find complement of the graph G where

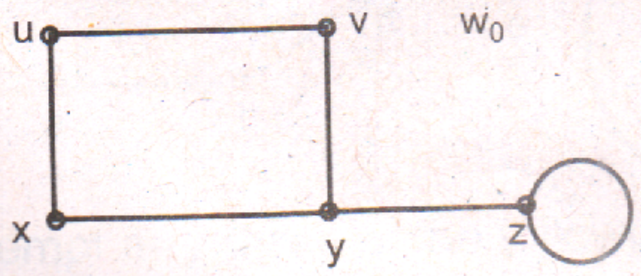


G

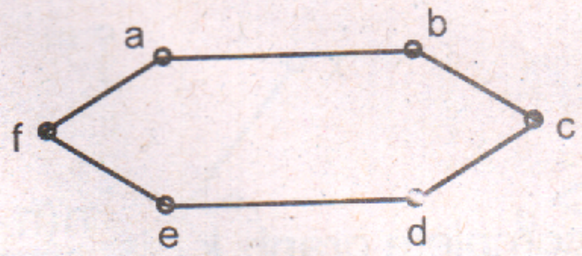
   3) Define a trial.
   4) What is a cut vertex in the graph.
   5) State hand shaking lemma.
   6) Draw any one Kuratowski's graph.
   7) Define a balanced digraph.
   8) Define a rooted tree.

12

2. Attempt **any six** of the following.
   1) Verify Hand shaking lemma for the graph.

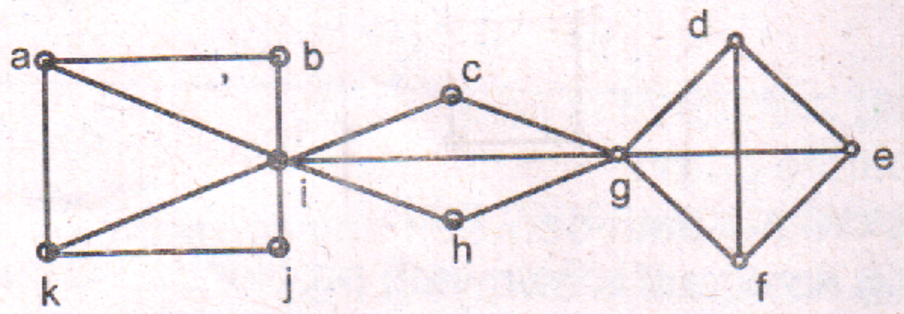


   2) Draw any two non isomorphic spanning subgraphs of the graph.



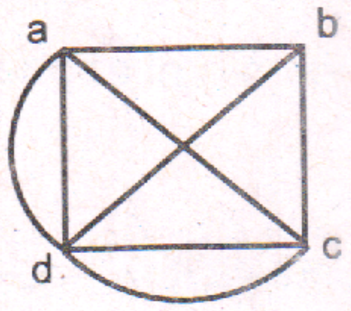
   3) Draw the complete bipartite graph which is not regular.

4) For the following graph with usual notations determine values of K(G) & λ(G).



5) Draw the graphs which are
   a) Both Eulerian & Hamiltonian
   b) Neither Eulerian nor Hamiltonian

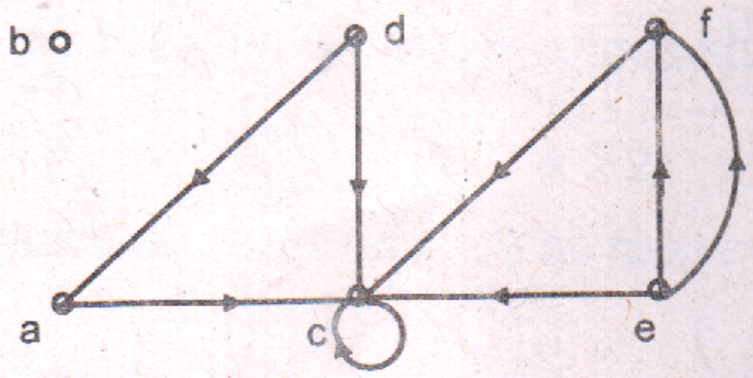
1. 6) Find the no. of faces of the graph.



7) Find rank and nullity of the graph k_5.

8) Draw binary tree having 11 vertices.

9) Find indegree & out degree of each vertex of the following graph.

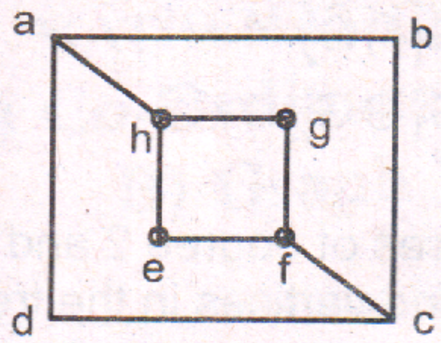


and verify that $\sum_{V \in V(G)} d^+(v) = \sum_{V \in V(G)} d^-(v)$.

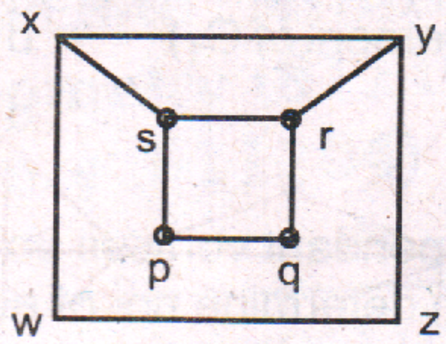
3. Attempt **any four** of the following.

1) Prove that total no. of edges in a complete graph k_n is $\frac{n(n-1)}{2}$.

2) Are the following graphs are isomorphic? Justify.



G_1

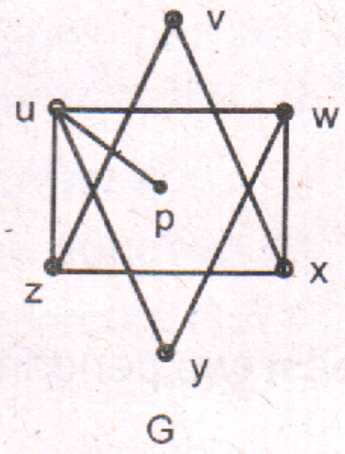


G_2

3) Prove that $k_{3,3}$ is not planer.

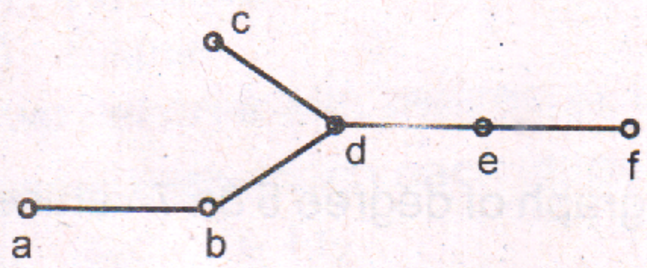
4) Prove that if a graph has exactly two odd vertices then there exist a path in the graph joining these two odd vertices.

5) Find geometrical dual of the graph.



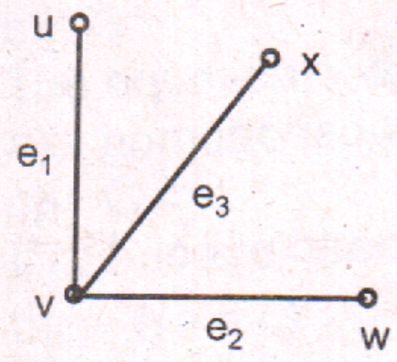
G

6) Find centre, radius and diameter the graph.

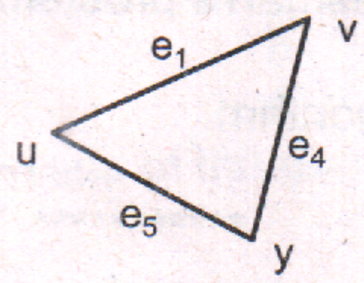


4. Attempt **any three** of the following. **12**

1) For the following graphs G_1 & G_2.



G_1



G_2

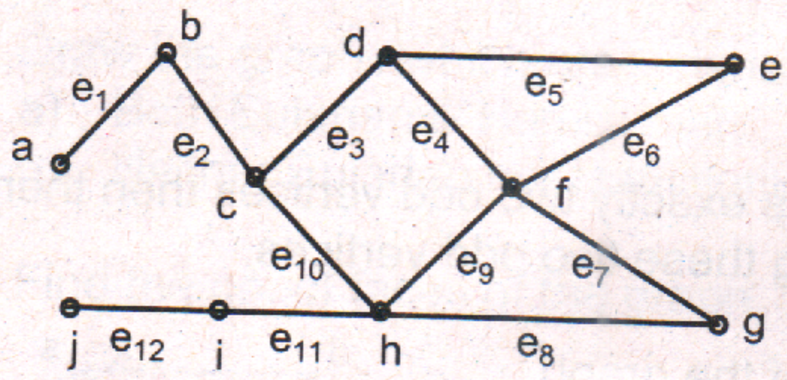
Find

a) $\overline{G_1}$ (complement of G_1) b) $G_1 \cap G_2$

c) $G_1 \cup G_2$ d) $G_1 \oplus G_2$

2) A tree has 2n pendant vertices 3n vertices of degree 2 and n vertices of degree then determine no. of edge and vertices in the tree.

3) In the following graph G



G

a) Give any three bridges
b) Give any three cut vertices
c) Write a path of length 8
d) Write a cycle of length 6

4) Prove that a non trivial tree T must have at least two pendant vertices.

5) Using colouring of the graphs colour the graphs $k_{4,3}$ and k_6 hence obtain their chromatic no.

5. Attempt **any two** of the following. **12**

1) a) Does there exist a regular graph of degree 5 on 7 vertices? Justify

b) Suppose a Tiger eats cow, dog and cat, Cow eats grass. Dog eats rabit, cat and mouse. Cat eats mouse, milk and chapati. Mouse eats chapati, Rabit eats grass then draw the flow chart of food cycle of these animals.

2) Explain Travelling salesman's problem.

3) Explain Warshall's Algorithm.

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